Inflation-Hedging Properties of Real Assets and Implications for Asset–Liability Management Decisions

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A recent surge in worldwide inflation has increased investors' awareness of the need to hedge against unexpected changes in price levels. Despite the current credit crisis and consequential economic slowdown that have somewhat eased the inflation concern for the short term, increasing inflation is a trend that is likely to continue for the foreseeable medium- to long-term future given the long-term increased demand pressure on food and energy resources. Inflation hedging is particularly important for pension funds facing pension payments that are indexed (conditional or full indexation) with respect to consumer price or wage level indices. A variety of cash instruments, such a Treasury inflation-protected securities (TIPS), as well as dedicated OTC derivatives, such as inflation swaps, are typically used to tailor customized inflation exposures that are suited to each particular pension fund liability profile. One outstanding problem, however, is that the capacity of the inflation-linked-securities market is not sufficient to meet the collective demand of institutional and private investors, while the OTC inflation derivatives market suffers from a perceived increase in counterparty risk. In addition, real returns on inflation-protected securities, negatively impacted by the presence of a significant inflation risk premium, are typically very low, which implies that investing in inflation-linked securities, when feasible, is a costly option for pension funds and their sponsor companies.

In this context, it has been argued that some other asset classes, such as stocks and nominal bonds, but also real estate or commodities, could provide useful, albeit imperfect, inflation protection at a lower cost compared to investing in TIPS. On the one hand, equity investments appear to be relatively poor inflation-hedging vehicles from a short-term perspective. Empirical evidence suggests that there is a negative relationship between expected stock returns and expected inflation (see Fama and Schwert [1977], Gultekin [1983], and Kaul [1987], among others), which is consistent with the intuition that higher inflation leads to lower economic activity, thus depressing stock returns (e.g., Fama [1981]). On the other hand, higher future inflation leads to higher dividends and thus higher returns on stocks (Campbell and Shiller [1988]), and thus equity investments should offer significant inflation protection over longer horizons, a fact that has been confirmed by a number of recent empirical academic studies (Boudoukh and Richardson [1993] and Schotman and Schweitzer [2000]). This property is particularly appealing for long-term investors, such as pension funds, who need to match increases in price level at the horizon, but not necessarily on a monthly basis. Similar inflation-hedging properties are expected for bond returns. Indeed, bond yields may be decomposed into real yield and expected inflation components. Because expected and realized inflation move together in the long term (see Schotman and Schweitzer [2000]), we expect a positive long-term correlation between bond returns and changes in inflation. In the short term, however, expected inflation may deviate from the actual realized inflation, leading to low or negative correlations. An investor willing and able to relax short-term constraints to focus on long-term inflation-hedging properties will find that investing in nominal bonds can provide a cost-efficient alternative, or complement, to investing in inflation-linked securities.

Moving beyond traditional investment vehicles, such as stocks and bonds, recent academic research has also suggested that alternative forms of investments offer attractive inflation-hedging benefits. Commodity prices, in particular, have been found to be leading indicators of inflation in that they are quick to respond to economy-wide shocks to demand. Commodity prices generally are set in highly competitive auction markets and consequently tend to be more flexible than prices overall. Recent inflation has been heavily driven by the increase in commodity prices, in particular, in the domain of agriculture, minerals, and energy. Consistent with these theoretical arguments, a recent study by Gorton and Rouwenhorst [2006] found that over the 1959–2004 period commodity futures were positively correlated with inflation, unexpected inflation, and changes in expected inflation. They also found that inflation correlations tend to increase with the holding period and are larger at return intervals of one and five years than at the monthly or quarterly frequency. Commercial and residential real estate also provide at least a partial hedge against inflation, which implies that portfolios that include real estate allow for enhanced inflation-hedging benefits (see Fama and Schwert [1977], Hartzell, Heckman, and Miles [1987], or Rubens, Bond, and Webb [1989]). This effect seems to be particularly significant over long horizons. Hence, Anari and Kolari [2002] examined the long-run impact of inflation on homeowner equity by investigating the relationship between house prices and the prices of non-housing goods and services, rather than return series and inflation rates, and inferred that house prices are a stable inflation hedge in the long run. The implications of such findings for asset–liability management (ALM) have been discussed by Hoevenaars, Molenaar, Schotman, and Steenkamp [2008], who constructed optimal mean-variance portfolios with respect to inflation-driven liabilities using the vector-autoregressive (VAR) approach from the literature on return predictability (see Kandel and Stambaugh [1996], Campbell and Viceira [1999], and Barberis [2000], among others).

In this article, we complement the existing literature on inflation-hedging properties of real assets, such as commodities and real estate, and the implications for asset–liability management by introducing an error correction form of the vector-autoregressive model (VECM), or cointegrated VAR model, which explicitly distinguishes between short-term and long-term dynamics in the joint distribution of asset returns and inflation. Although the error correction form of the vector-autoregressive model has been extensively used in the macroeconomic literature to distinguish between trends and business cycles, and thus between stationary and nonstationary components in consumption and wealth dynamics (see, e.g., Lettau and Ludvigson [2004] or Beaudry and Portier [2006]), our article is, to the best of our knowledge, the first to provide a comprehensive VECM model for the formal analysis of inflation-hedging properties of various traditional and alternative classes.[1]

Moreover, the article provides a structural form of the model that incorporates i.i.d. innovations, allowing for the generation of a stochastic Monte Carlo analysis in a straightforward manner. Our results suggest that novel long-term liability-hedging investment solutions can be designed to decrease the cost of inflation insurance from the investor's perspective. In particular it is possible to construct enhanced versions of inflation-hedging portfolios that involve investing in inflation-linked securities as well as commodities and real estate in order to achieve satisfactory levels of inflation hedging over the long-term at a lower cost compared to a solution solely based on inflation swaps. The intuition behind these results is straightforward. The increased expected return potential generated through the introduction of commodities and real estate in addition to TIPS in the liability-hedging portfolio (LHP) allows for a reduced global allocation to the performance-seeking portfolio (PSP) while meeting the global performance expectations that, in turn, allows for better risk management properties.

MODELING RETURN AND INFLATION DYNAMICS

The first key challenge that needs to be met for the analysis of the benefits of alternative investment strategies from an asset–liability perspective is the design of an appropriate econometric model for the joint distribution of asset returns and inflation. The bulk of the literature on stock return predictability and return dynamics modeling (e.g., Campbell and Shiller [1988]) has relied on VAR models. We extend this literature by introducing an error correction form of the vector-autoregressive model, or cointegrated VAR model, which explicitly accounts for the presence of long-term *cointegration* relationships between asset prices and the levels of various state variables including inflation. Two or more series are said to be cointegrated if a linear combination of the series is stationary while the series are themselves nonstationary (see Engle and Granger [1987]). Cointegrated relationships have been found to hold between consumption and income as in Davidson, Hendry, Srba, and Yeo [1978] or between stock earnings or dividends and stock prices (Campbell and Shiller [1988]). In both cases, the amount of the deviation from long-term equilibria has been proven to yield substantial predictive power for the variables in the system. To account for the presence of such long-term relationships in price series, the VAR model can be generalized through the following error correction form (VECM):

$$\Delta y_t = c + \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \cdots + \Gamma_p \Delta y_{t-p} + u_t \qquad (1)$$

where y_t represents a $n \times 1$ vector of endogenous variables, c is a vector of constants, Γ_i are $n \times n$ coefficient matrices and u_t is the innovation process. The reduced rank matrix Π can be decomposed into $\Pi = \alpha\beta'$. Its rank $r < n$ determines the number of linear-independent long-term equilibrium relationships and is also called *cointegration rank*. In other words, there are r independent linear combinations of the lagged endogenous variables that define the cointegration relationships constituting r stationary variables, $\beta' y$. Accordingly, β is a $n \times r$ matrix that hosts the cointegrating vectors so that $\beta' y_t$ is stationary and reflects the long-term equilibrium relationships of the variables, while α is a $1 \times n$ vector that hosts the corresponding adjustment parameters, that is, the parameters that determine the reversion speed to this long-term equilibrium.

In the context of cointegrated processes, the dynamics of the underlying variables may be separated into short-run and long-run dynamics. Short-run dynamics are driven by the structural responses to lagged innovations captured by Γ_i in Equation (1). The cointegration relationship vector, β, and the reversion speed vector, α, govern the long-run dynamics. More precisely, $\Pi = \alpha\beta'$ induces instantaneous shocks to the system if it deviates from the long-term equilibrium. If the system is cointegrated, cross-sectional responses to shocks may be nontransitory or persistent as the time series "hang together."

We will now introduce a modeling approach that uses long-run and short-run restrictions in order to identify the structural shocks of the system. The structural form of the model is characterized by i.i.d. innovations, $\varepsilon_{t,i}$, as opposed to the correlated original innovation process, $u_{t,i}$. For this, we search for the transformation matrix B such that

$$u_t = \mathrm{B}\varepsilon_t \qquad (2)$$

From the structural assumption we know that the covariance matrix of the innovations ε (Σ_ε) is diagonal. Without any loss of generality we postulate Σ_ε to be the identity matrix. Therefore, the original innovation covariance matrix may be written as $\Sigma_u = \mathrm{BB}'$. The transformation matrix B hosts $n \times n$ parameters that need to be identified. Since Σ_u is symmetric, only $\frac{1}{2}n(n+1)$ independent equations are available from $\Sigma_u = \mathrm{BB}'$. For the parameters to be identified we need $\frac{1}{2}n(n-1)$ additional restrictions. The matrix B can accordingly be estimated by the maximum likelihood method. Following Breitung, Brüggemann, and Lütkepohl [2004], the corresponding log-likelihood function is given by[2]

$$l(\mathrm{B}) = \text{const} - \frac{T}{2}\log|B|^2 - \frac{T}{2}\mathrm{tr}((\mathrm{B}')^{-1}\mathrm{B}^{-1}\Sigma_u) \qquad (3)$$

Once the transformation matrix is identified, we can write the Structural VECM (SVECM) in its reduced form,

$$\Delta y_t = c + \Pi y_{t-1} + \Gamma \Delta y_{t-1} + \mathrm{B}\varepsilon_t \qquad (4)$$

We will use the i.i.d. innovation-process property to perform a Monte Carlo analysis based on the fitted model. The generated scenarios will subsequently be exploited in a portfolio construction context.

DATA AND TERM STRUCTURE OF RISK

Our empirical analysis focuses on a set of traditional and alternative asset classes. Stock returns are represented by the CRSP value-weighted stock index. Commodities are proxied by the S&P Goldman Sachs Commodity Index (GSCI). Real estate investments are represented by the FTSE NAREIT Index, which is a value-weighted basket of REITs listed on the NYSE, AMEX, and NASDAQ. We thus limit the opportunity set to liquid and publicly traded assets. Finally, we add the Lehman Long U.S. Treasury Index, as well as the one-month Treasury bill rate.[3] Following the evidence from the extensive literature on return predictability (see Stock and Watson [1999], among others), we also add potential predictive economic variables to the set of endogenous variables. We introduce the dividend yield (see, e.g., Campbell and Shiller [1988], Hodrick [1992], or Campbell and Viceira [2002]), the credit spread (computed as the difference between Moody's Seasoned Baa Corporate Bond Yield and the 10-Year Treasury Constant Maturity Rate), and the term spread (obtained from the difference between the 10-Year Treasury Constant Maturity Rate and the one-month T-Bill rate). The dividend yield data is obtained from CRSP and all other economic figures were obtained from the U.S. Federal Reserve Economic Database.[4] Our analysis is based on quarterly returns from Q1 1973 through Q4 2007.

In terms of the liabilities, we include an inflation proxy represented by the consumer price index (CPI). As in Hoevenaars et al. [2008], we focus on a pension fund that is in a stationary state as would be the case when the age cohorts and the built-up pension rights per cohort are constant through time. Under this assumption, and further assuming that liability payments exhibit unconditional inflation indexation, the return on the liability portfolio can be proxied by the return on a constant maturity zero-coupon TIPS with a maturity equal to the duration of the liability cash flows.[5] We construct the time series for such constant maturity zero-coupon TIPS in accordance with the methodology described in Kothari and Shanken [2004], which states that the nominal return on a real bond is given as the sum of a real yield plus realized inflation. The real yield is in turn obtained as the difference of the nominal yield and the sum of expected inflation plus the inflation risk premium. As in Kothari and Shanken [2004], we assume the inflation risk premium to be equal to zero.[6] We simplify the computation of expected inflation and use 60-month moving-average inflation. As a result, we obtain the returns on liabilities as

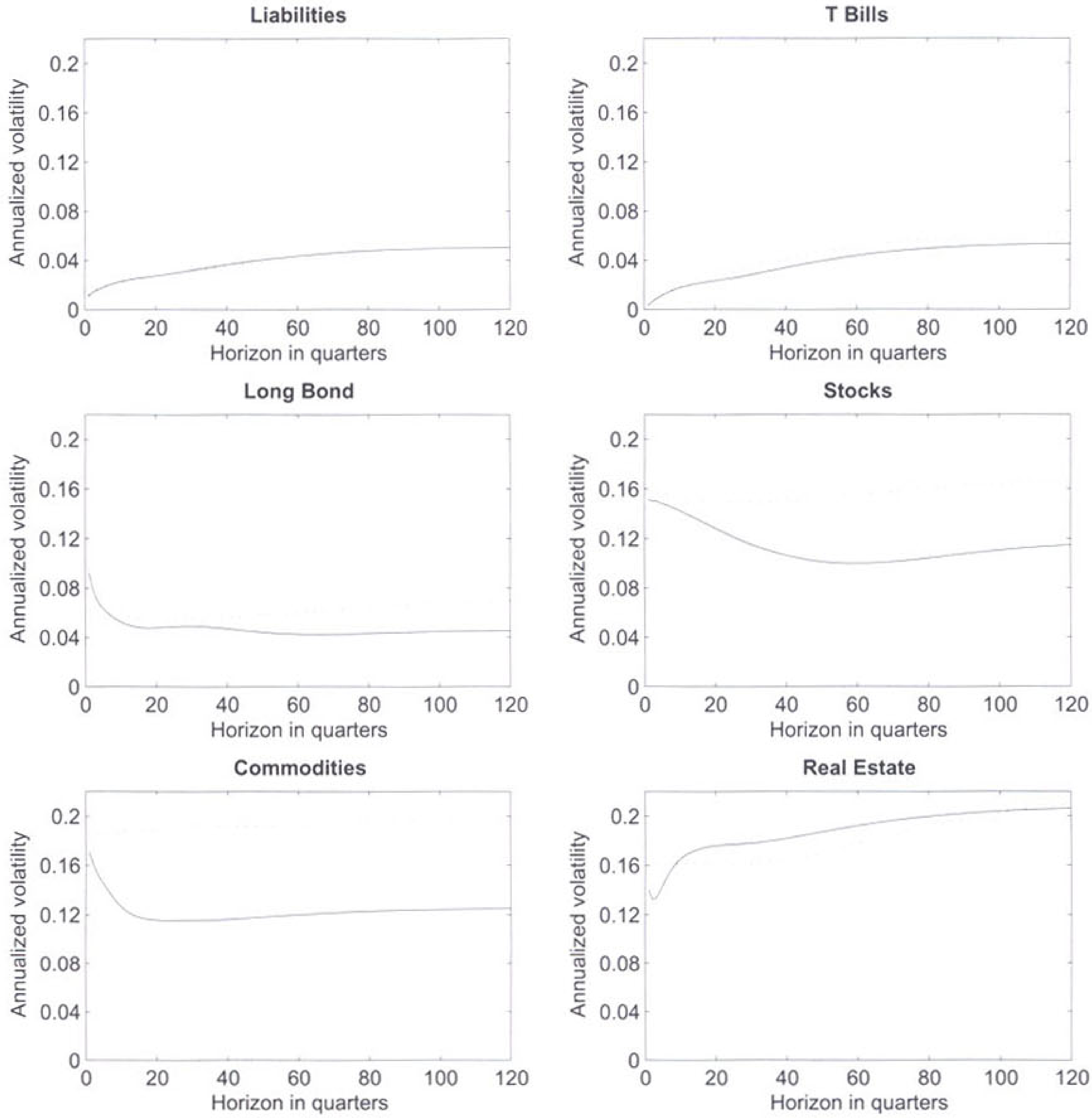
$$r_{L,t} = \text{yield}_t^{(\tau)} - \text{E}_t(\pi) \tag{5}$$

where the upper index τ indicates the duration of the liabilities that we have arbitrarily chosen in what follows to be equal to 20 years.[7]

In Exhibit 1, we plot the annualized volatilities of returns on liabilities and asset classes for different investment horizons according to the equations derived in the appendix. A particular focus of the graphs is on the difference between VAR-implied volatilities (dashed lines) and VECM-implied volatilities (solid lines). The differences between the two econometric methodologies are rather significant for bonds, stocks, and commodities, with VECM-implied volatilities proving to be significantly lower than VAR-implied volatilities for these classes. Liability, T-bill, and real estate returns only show minor differences between the VAR and the VECM approaches. More specifically, VAR-implied volatilities seem to indicate that assets become more risky as the investment horizon increases, while VECM volatilities have contrasting implications for the various assets. Liabilities, T-bills, and real estate investments appear to be more risky in the long run, while bonds, stocks, and commodities exhibit a downward-sloping volatility structure, especially from very short- to medium-term horizons. The contrasted results in implied volatility estimates obtained with the VAR versus VECM model are due to the equilibrium-reverting character of the additional part $\alpha\beta' y_{t-1}$. As explained, although $\beta' y_{t-1}$ establishes the equilibrium relationship, α determines the instantaneous impact of a deviation from this equilibrium on Δy_t (see Equation (1)).

Next, Exhibit 2 displays horizon-dependent correlation coefficients between liability returns and the return on various asset classes. The plots clearly suggest that bond, stock, and real estate returns are negatively correlated with liabilities in the short run, and that the correlation coefficient exhibits an upward-sloping pattern as the investment horizon increases. Bond and stock returns start to become positively correlated with liability returns after about 60 quarters (15 years), ending with a significant, positive correlation of roughly 0.4 with a 30-year investment horizon. Again, the result allows us to identify significant discrepancies between VAR and VECM models, especially in the case of commodities and real estate. Commodity returns are positively correlated with liability returns in both cases, but the VECM implies a significantly higher and more stable correlation than the VAR. Model-implied correlations between real estate and liability returns, on the contrary, are significantly higher in the VAR model than what they are in the VECM. This may be due to the fact that commodities are part of all long-term equilibrium relationships, a result of the normalization process of the matrix β and the variable order permutation

EXHIBIT 1
Annualized Volatilities



Note: Dotted lines correspond to VAR-implied volatilities and solid lines to VECM-implied volatilities.

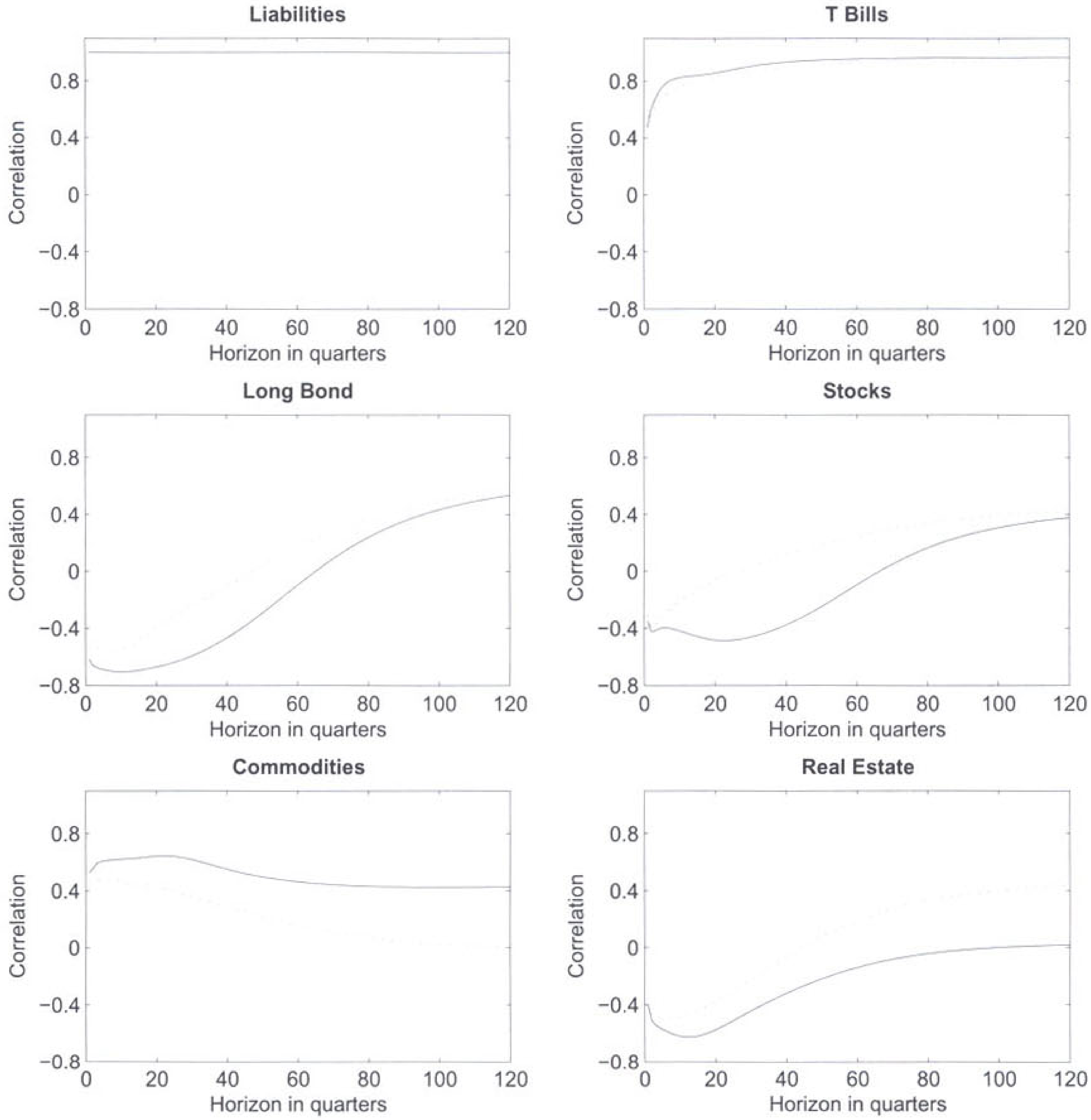
procedure previously described. In the next section, we will examine the impact of these model-implied moments and co-moments from a liability-hedging portfolio perspective.

INFLATION-HEDGING PROPERTIES OF VARIOUS PORTFOLIOS

Investment-horizon-dependent allocation decisions have been widely studied in the literature over the last decade (Brandt [2005] and Campbell and Viceira [2005]). The term structure of risk, merely driven by the presence of mean-reversion effects, with different speeds of mean reversion (Lettau and Wachter [2007]), also plays a central role in asset allocation decisions in the presence of liabilities (see also Campbell and Viceira [2005] regarding the term structure of risk). This section uses VECM model-implied dynamics to assess inflation and liability-hedging potential across different investment horizons.

In a framework where liabilities are indexed with respect to inflation, and when short-term liability risk

EXHIBIT 2
Correlation between Liabilities and Asset Returns



Note: Dotted lines correspond to VAR-implied correlations and solid lines to VECM-implied correlations.

hedging is the sole focus, the optimal liability-hedging portfolio (LHP) allocation consists of investing 100% in the inflation-indexed bond portfolio (TIPS portfolio), which unfortunately leads to very limited upside potential. Consequently, the investor needs a relatively sizable significant allocation to the highest risk–reward performance-seeking portfolio (PSP) in order to meet the return requirements, which in turn generates a relatively high funding risk. Intuitively, we expect that relaxing the constraint of a perfect liability fit for the LHP at the short-term horizon would allow one to include alternative asset classes in the LHP, leading to increased upside potential. Overall, this would allow an investor to reduce her allocation in the PSP, thus leading to a reduced surplus risk. To formalize this intuition, we perform a scenario-based analysis to derive the funding ratio distribution at various investment horizons. The data-generating process is described by the VECM. We further use the structural model to disentangle the correlated innovation process and transform it into i.i.d. innovations. We draw i.i.d. random variables from the multivariate standard normal distribution for the structural innovations $\varepsilon_t^s (s = 1 \ldots S)$ and obtain the modeled returns by

$$\Delta y_t^s = c + \Pi y_{t-1}^s + \Gamma \Delta y_{t-1}^s + B\varepsilon_t^s \tag{6}$$

for a total of $S = 5{,}000$ simulated paths. The first variable in y_t^s represents the liability return. We evaluate the different portfolios in terms of the funding ratio (FR) distribution.

The funding ratio at t in scenarios s is accordingly given by

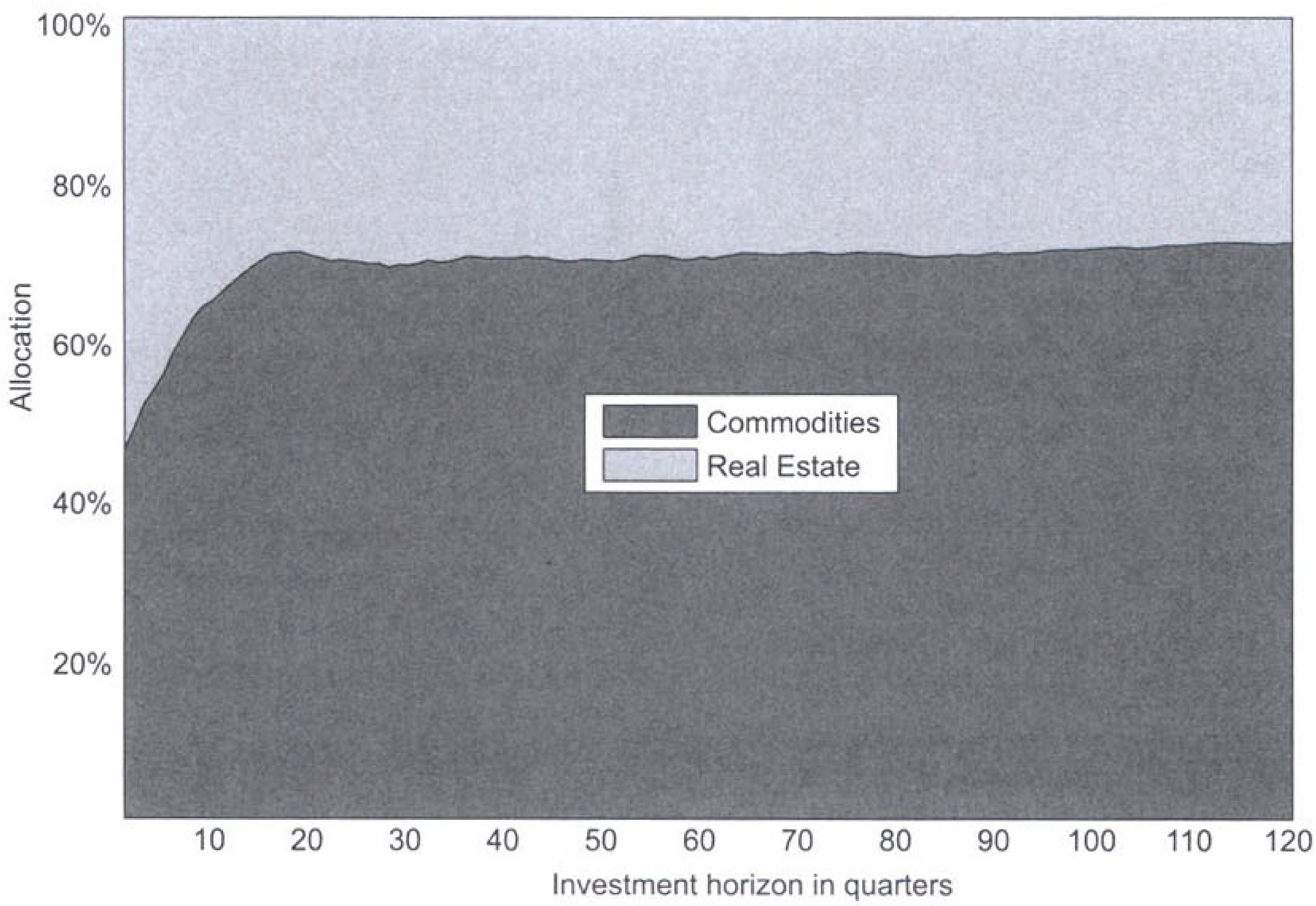
$$FR_t^s = \exp((\omega' - \iota) y_t^s) \tag{7}$$

where ι denotes the $n \times 1$ vector containing a 1 in the first position and zeros elsewhere, and ω is the portfolio vector.

The encouraging correlation and volatility implications studied in the previous section, taken together with the finding that alternative assets typically yield higher returns than TIPS, seem to suggest that introducing commodities and real estate, in addition to TIPS, in a pension fund LHP allows for upside potential while limiting shortfall probabilities to a reasonably low level, at least from a long-term perspective. In what follows, we quantify the trade-off between a deviation from the perfect liability match and the resulting return upside potential, which has the welcome side effect of decreasing the required contributions. The consequences, in terms of ALM risk budgets, of introducing alternative asset classes in order to design enhanced liability-hedging portfolios with improved performance will be quantitatively analyzed in a subsequent section.

In order to analyze the characteristics of liability-hedging portfolios that are enhanced by commodities and real estate assets, we will proceed in two steps. First, we find the optimal portfolio mix of commodities and real estate, and second, we add this portfolio to TIPS in various proportions to form the enhanced liability-hedging portfolio. The first step is addressed by finding the portfolio of commodities and real estate that minimizes the tracking error volatility with the TIPS portfolio. Exhibit 3 shows the

EXHIBIT 3
Alternative LHP Enhancers



Note: The allocations correspond to those that minimize the tracking error with TIPS, defined as the standard deviation between the portfolio and the TIPS return, for a given investment horizon. Optimization is based on the 5,000 simulated scenarios.

resulting portfolios as a function of the investment horizon. The portfolio is well balanced between the two assets, and the position in commodities increases with the investment horizon.

Accordingly, we enhance the TIPS LHP by fractions of this alternative portfolio made up of commodities and real estate. Exhibit 4 presents funding ratio statistics for various enhanced LHPs, ranging from 0% to 50% in alternative investments (AI) with the remainder in TIPS. In particular, Panel A of Exhibit 4 shows that the upside potential is an increasing function of the percentage allocated to alternative assets within the LHP portfolio. In contrast, the more the investor allocates to alternative assets, the higher the risk to fall short of the liabilities (Panel B). Panel C indicates that the probability to fall severely short is very low. Even for an investment to the AI portfolio of 50%, severe shortfall probabilities are only 6% in the short run and 2% for long investment horizons. For modest investments to the AI portfolio (0%–15%), the severe shortfall probability even decreases to 0%, meaning that none of the 5,000 simulated paths yields a funding ratio lower than 90%, regardless of the

Exhibit 4
Enhanced LHP: ALM Indicators

Panel A: Mean Funding Ratio

	Allocation to AI (in %)										
Horizon	0	5	10	15	20	25	30	35	40	45	50
3	1.00	1.01	1.01	1.02	1.02	1.03	1.04	1.04	1.05	1.05	1.06
7	1.00	1.01	1.03	1.04	1.06	1.07	1.09	1.10	1.11	1.13	1.14
10	1.00	1.02	1.04	1.06	1.09	1.11	1.13	1.15	1.17	1.19	1.22
15	1.00	1.04	1.07	1.11	1.14	1.18	1.22	1.25	1.29	1.33	1.36
20	1.00	1.05	1.11	1.16	1.22	1.27	1.33	1.38	1.44	1.49	1.55
30	1.00	1.11	1.22	1.33	1.44	1.55	1.66	1.77	1.88	1.99	2.10

Panel B: Probability of Shortfall (FR < 1.0)

	Allocation to AI (in %)										
Horizon	0	5	10	15	20	25	30	35	40	45	50
3	0.00	32.78	32.78	32.78	32.78	32.78	32.78	32.78	32.78	32.78	32.78
7	0.00	22.30	22.30	22.30	22.30	22.30	22.30	22.30	22.30	22.30	22.30
10	0.00	17.48	17.48	17.48	17.48	17.48	17.48	17.48	17.48	17.48	17.48
15	0.00	12.98	12.98	12.98	12.98	12.98	12.98	12.98	12.98	12.98	12.98
20	0.00	9.08	9.08	9.08	9.08	9.08	9.08	9.08	9.08	9.08	9.08
30	0.00	5.20	5.20	5.20	5.20	5.20	5.20	5.20	5.20	5.20	5.20

Panel C: Probability of Severe Shortfall (FR < 0.9)

	Allocation to AI (in %)										
Horizon	0	5	10	15	20	25	30	35	40	45	50
3	0.00	0.00	0.00	0.00	0.00	0.10	0.56	1.84	3.28	4.42	5.84
7	0.00	0.00	0.00	0.00	0.08	0.44	1.48	2.68	3.88	5.00	6.06
10	0.00	0.00	0.00	0.00	0.10	0.56	1.58	2.56	3.48	4.32	5.36
15	0.00	0.00	0.00	0.00	0.24	0.78	1.62	2.28	3.10	3.72	4.20
20	0.00	0.00	0.00	0.00	0.10	0.52	0.94	1.56	2.10	2.60	3.10
30	0.00	0.00	0.00	0.00	0.20	0.46	0.76	0.98	1.26	1.64	1.92

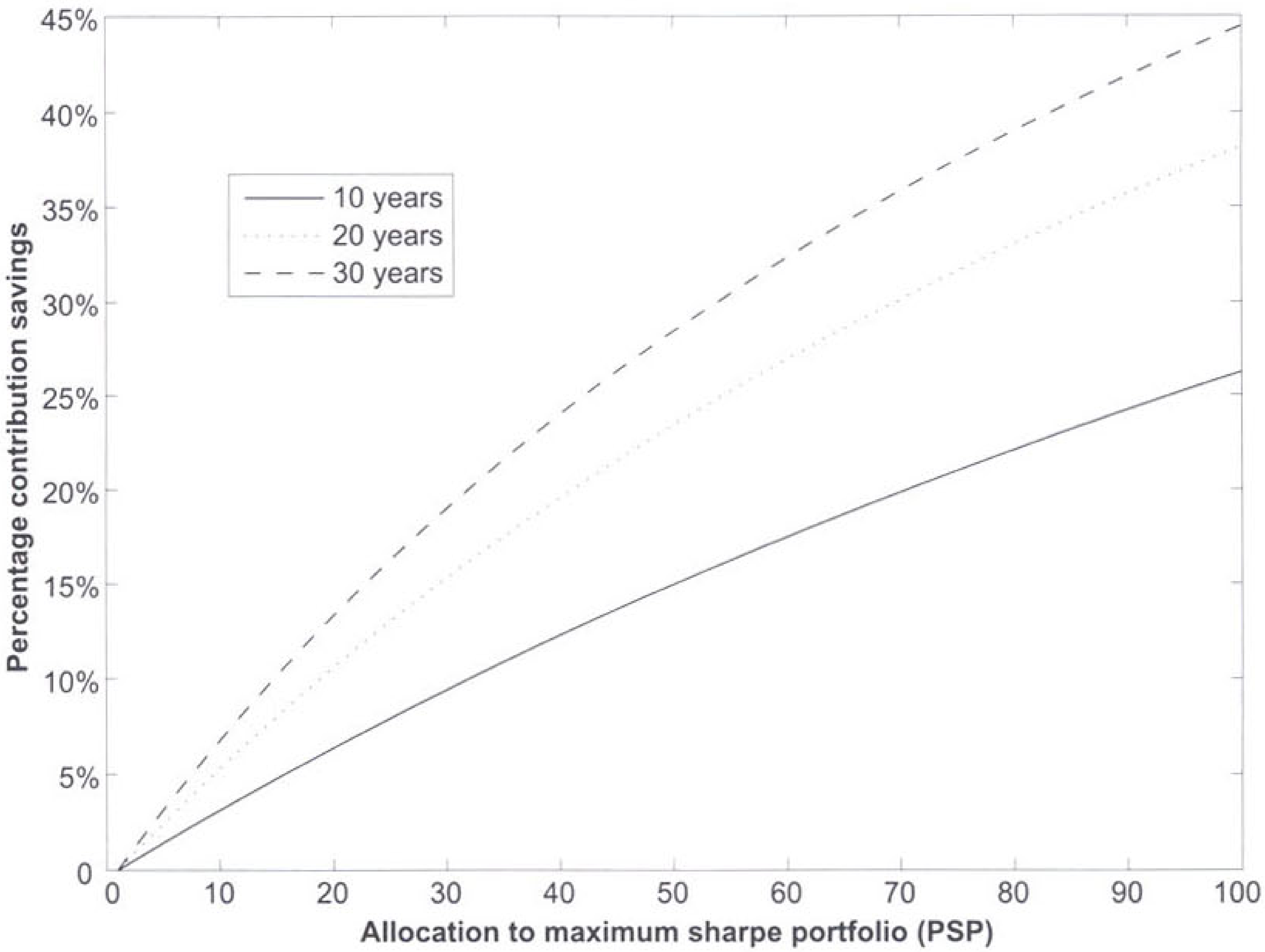
Note: Liability-hedging capacity is examined for portfolios made up of the LHP (TIPS) and the alternative investment (AI) portfolio of commodities and real estate that maximizes the correlation with TIPS (see Exhibit 3). Mean funding ratios and shortfall probabilities (in percent) for various investment horizons (in years) and based on 5,000 simulated scenarios are given.

investment horizon. Overall these results suggest that the introduction of alternative investment vehicles may lead to increased upside potential for the LHP without severely increasing the shortfall risk.

IMPLICATIONS FOR ASSET–LIABILITY MANAGEMENT

In terms of risk budgets, the implementation of liability-driven investment solutions critically depends on the attitude toward risk. It is typically understood that high risk-aversion levels lead to a predominant investment in the liability-hedging portfolio, which implies low extreme funding risk (zero risk in the complete market case), as well as low expected performance and, therefore, high necessary contributions. Low risk aversion levels, however, lead to a predominant investment in the performance-seeking portfolio, which implies high funding risk as well as higher expected performance and, hence, lower contributions. To formalize this intuition, we compare the initial contribution that is needed to generate a 100% funding ratio at the horizon when the investor's portfolio is fully invested in TIPS (the perfect liability-hedging portfolio) versus the initial contribution needed to generate an *average* 100% funding ratio at the horizon when risky asset classes, such as stocks and bonds, are introduced. Exhibit 5 presents a graphical representation of this effect for different investment horizons. For instance, for an investment horizon of 20 years, an allocation of 40% to the PSP (which is assumed to contain stocks and bonds in a proportion that generates the maximum Sharpe ratio) and 60% to the liability-hedging portfolio fully invested in TIPS allows the reduction of initial contributions by almost 20% compared to a 100% investment in the TIPS liability-hedging portfolio. Of course, this contribution savings effect comes at the cost of introducing funding

EXHIBIT 5
Contribution Savings through Introduction of a PSP



Note: Contribution saving is defined as the potential reduction of initial investment when deviating from the perfect liability-matching portfolio, such that, on average, the pension plan is fully funded. Numbers are based on 5,000 simulated scenarios. The PSP contains stocks and bonds in proportions that depend on the investment horizon.

Exhibit 6
Global Portfolio (LHP + PSP): ALM Indicators

Panel A: Mean Funding Ratio

	Allocation to PSP (in %)										
Horizon	0	5	10	15	20	25	30	35	40	45	50
3	1.00	1.00	1.01	1.01	1.02	1.02	1.03	1.03	1.04	1.04	1.05
7	1.00	1.01	1.03	1.04	1.05	1.06	1.08	1.09	1.10	1.12	1.13
10	1.00	1.02	1.04	1.06	1.08	1.09	1.11	1.13	1.15	1.17	1.19
15	1.00	1.03	1.05	1.08	1.10	1.13	1.16	1.18	1.21	1.23	1.26
20	1.00	1.03	1.06	1.10	1.13	1.16	1.19	1.22	1.26	1.29	1.32
30	1.00	1.04	1.08	1.12	1.16	1.21	1.25	1.29	1.33	1.37	1.41

Panel B: Probability of Shortfall (FR < 1.0)

	Allocation to PSP (in %)										
Horizon	0	5	10	15	20	25	30	35	40	45	50
3	0.00	38.00	38.00	38.00	38.00	38.00	38.00	38.00	38.00	38.00	38.00
7	0.00	32.34	32.34	32.34	32.34	32.34	32.34	32.34	32.34	32.34	32.34
10	0.00	29.38	29.38	29.38	29.38	29.38	29.38	29.38	29.38	29.38	29.38
15	0.00	23.62	23.62	23.62	23.62	23.62	23.62	23.62	23.62	23.62	23.62
20	0.00	19.20	19.20	19.20	19.20	19.20	19.20	19.20	19.20	19.20	19.20
30	0.00	12.94	12.94	12.94	12.94	12.94	12.94	12.94	12.94	12.94	12.94

Panel C: Probability of Severe Shortfall (FR < 0.9)

	Allocation to PSP (in %)										
Horizon	0	5	10	15	20	25	30	35	40	45	50
3	0.00	0.00	0.00	0.00	0.02	0.22	1.24	3.04	5.14	6.90	8.70
7	0.00	0.00	0.00	0.00	0.66	2.86	4.96	7.76	10.28	12.12	14.04
10	0.00	0.00	0.00	0.06	1.06	3.18	6.40	8.64	10.82	12.66	14.02
15	0.00	0.00	0.00	0.04	1.06	2.92	4.82	6.66	7.88	9.30	10.76
20	0.00	0.00	0.00	0.02	0.38	1.60	3.40	4.62	5.92	6.90	7.88
30	0.00	0.00	0.00	0.00	0.10	0.78	1.70	2.60	3.56	4.34	4.92

Note: Liability-hedging capacity is examined for portfolios made up of the LHP (TIPS) and the PSP (stocks and bonds, according to the allocation that maximizes Sharpe ratio). Mean funding ratios and shortfall probabilities (in percent) for various investment horizons (in years) and based on 5,000 simulated scenarios are given.

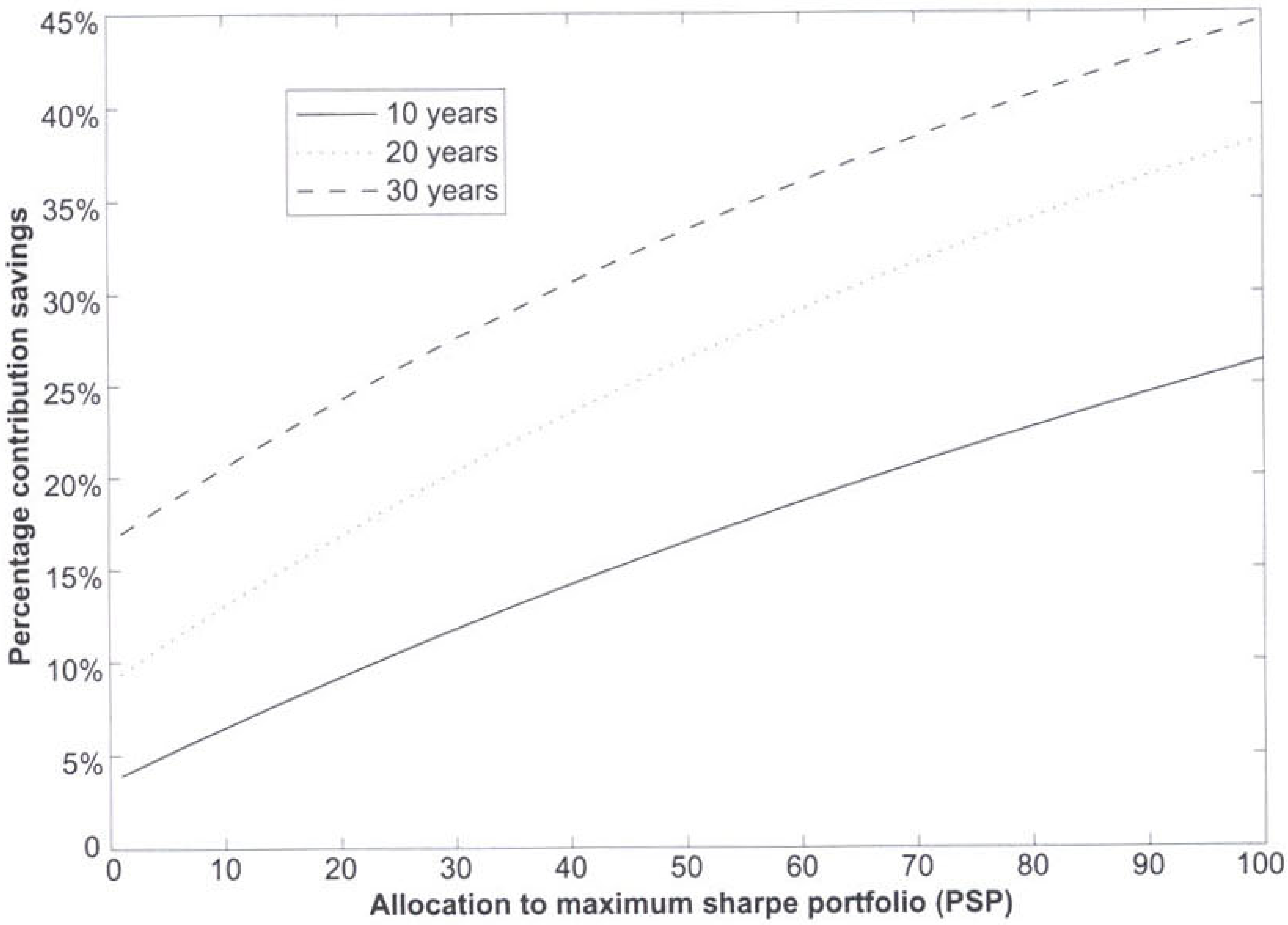
risk at the global portfolio level. As shown in Panel B of Exhibit 6, shortfall probabilities significantly increase with the allocation to the PSP, even though the magnitude of this effect decreases with the time horizon.

We now analyze the impact on ALM risk budgets of the introduction of real estate and commodities within the LHP. We first draw a comparison between the option that consists of investing 100% in the LHP, but enhancing the LHP with the introduction of real estate and commodities, and the option that consists in leaving the LHP, fully invested in TIPS and seeking to add performance potential through the introduction of the PSP, as previously discussed. A comparison of the results in Panels B and C of Exhibit 4 and the results in Panels B and C of Exhibit 6 clearly indicates that introducing alternatives within the LHP (Option 1) systematically leads to a lower increase in risk indicators compared to the introduction of traditional asset classes through the PSP (Option 2). For example, the probability of a shortfall greater than 90% at the 20-year horizon is 2.1% when the investor's portfolio is invested 60% in TIPS and 40% in the combination of real estate and commodities that allows for the best liability hedge (see Panel C of Exhibit 4), while it reaches 5.92% when the investor's portfolio is invested 60% in TIPS and 40% in the combination of stocks and bonds that allows for the maximum Sharpe ratio (see Panel C of Exhibit 6).

In the same spirit, Exhibit 7 shows the relative contribution savings as a function of the allocation to the PSP when the LHP is enhanced by 10% in alternative investments. In comparison to Exhibit 5, the graph suggests that for comparable allocations to the PSP, contribution savings are larger in magnitude when using enhanced LHP instead of the sole TIPS LHP. Consequently, the target contribution savings can now be reached with a lower allocation to the PSP portfolio. Exhibits 8 and 9 illustrate this effect for LHPs that are enhanced by the introduction of 5% and 10% respectively, of alternative investments (represented again by the portfolio of real estate and commodities that minimizes the tracking error with respect to the liability portfolio). For instance, with an LHP that is composed of 90% TIPS and 10% alternative assets (Exhibit 9), an allocation of only 27% to the PSP leads to the same mean funding ratio or, equivalently, to the same contribution savings as an investment of 40% in the PSP when the LHP is solely invested in TIPS.

Exhibit 10 presents the corresponding ALM risk indicators, with numbers that can be compared to the results in Exhibit 6. We observe that (for a given allocation to the PSP) enhanced LHPs do not only lead to higher mean funding ratios, but they also lead to lower shortfall probabilities (see Panel B of Exhibit 10) compared to the case of the non-enhanced (i.e., pure TIPS) LHP (Panel B of Exhibit 6). This is obviously related to portfolio diversification effects between traditional assets within the PSP and alternative assets within the LHP.

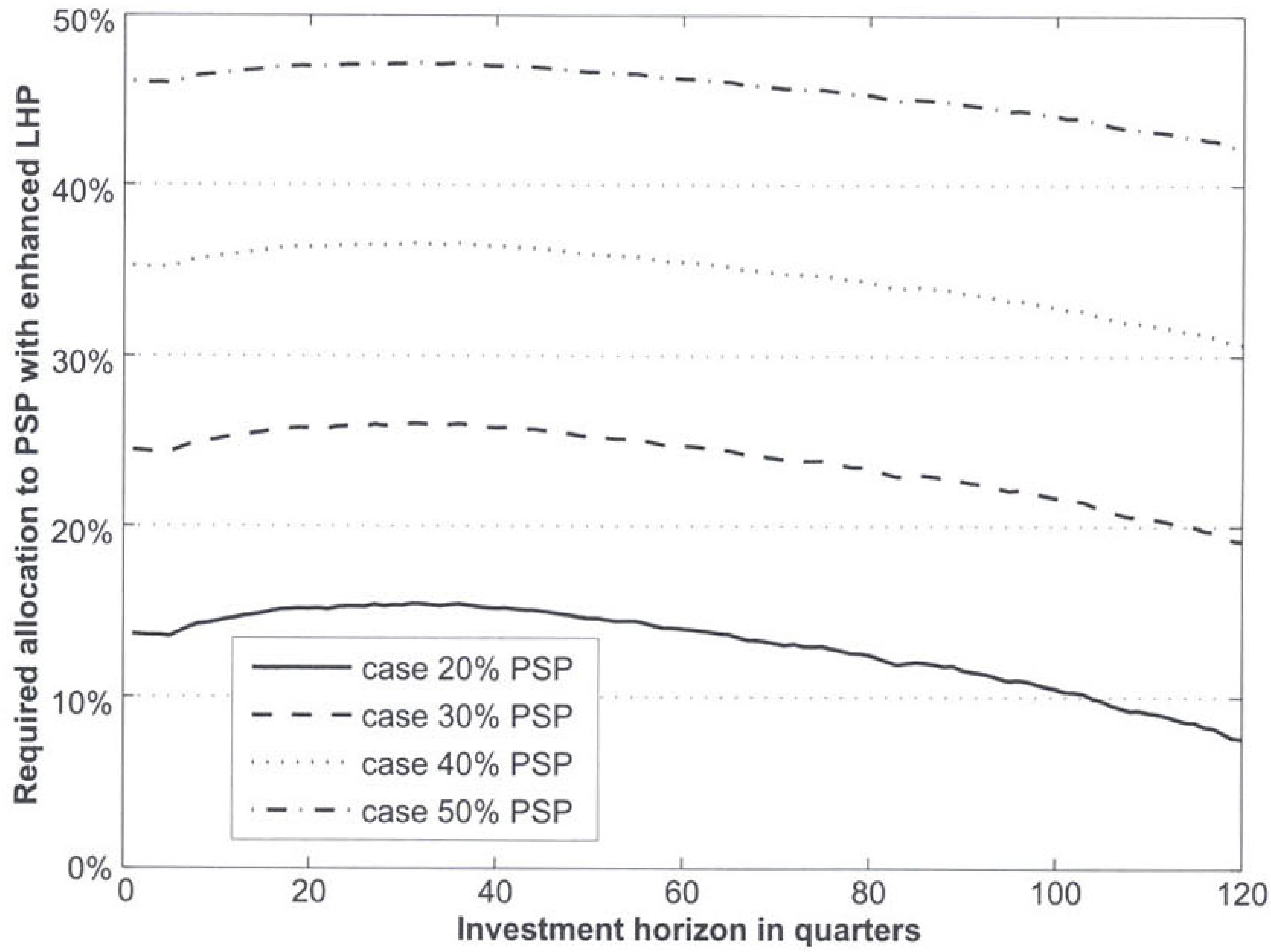
EXHIBIT 7
Contribution Savings through Enhanced LHP



Note: Contribution savings is defined as the potential reduction of the initial investment when deviating from the perfect liability-matching portfolio, such that, on average, the pension plan is fully funded. Numbers are based on 5,000 simulated scenarios. The alternative investment portfolio contains commodities and real estate in proportions that depend on the investment horizon (see Exhibit 3).

EXHIBIT 8

Return Equivalents with 5% AI: PSP Allocation Reduction



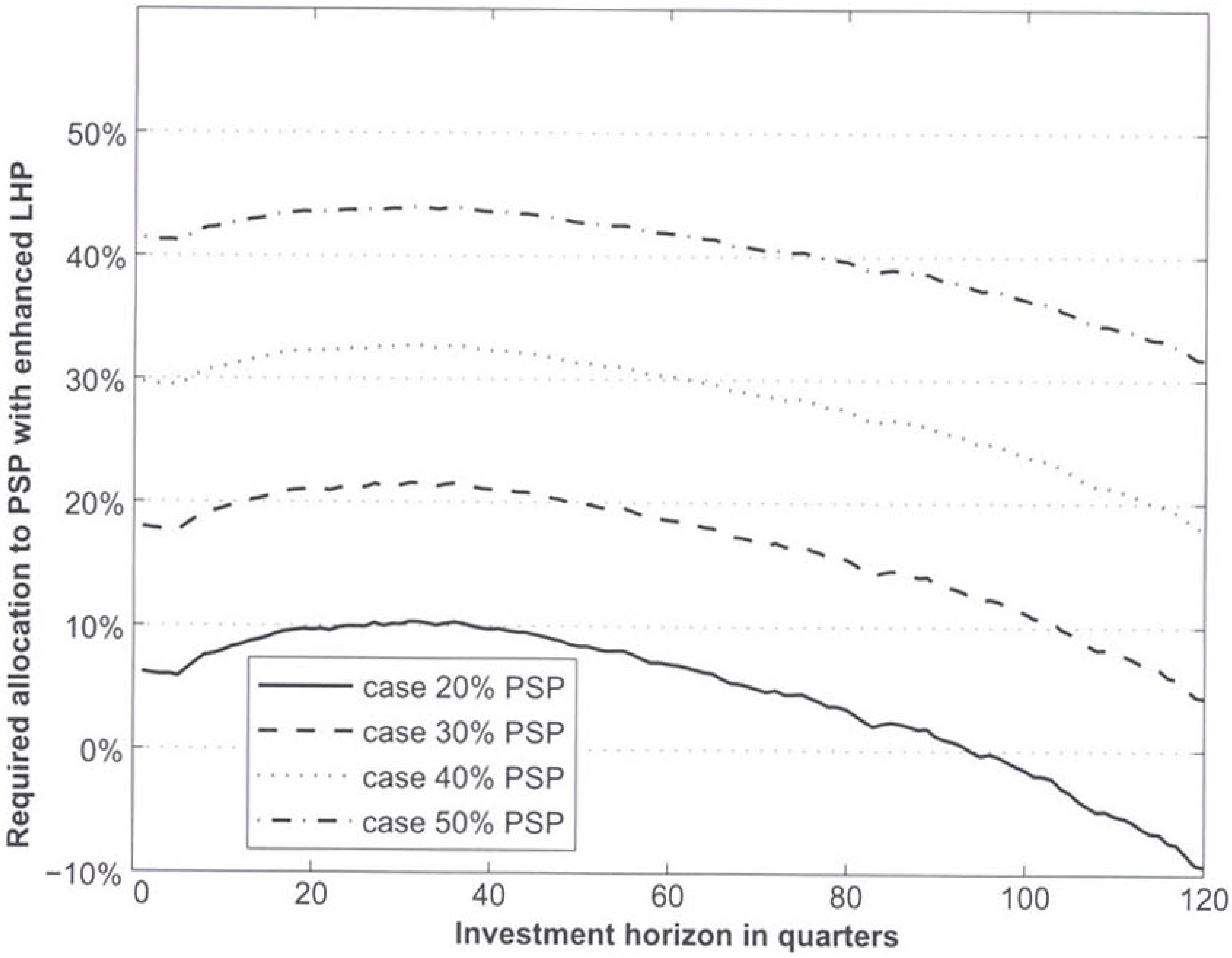
Note: The graph shows the impact of enhancing the LHP beyond TIPS. The enhanced LHP consists of 95% TIPS and 5% alternative investments (see Exhibit 3). The graph yields the required allocations to the PSP to obtain, on average, the same amount of return as in the case of the traditional LHP (TIPS). Various non-enhanced portfolios are taken as benchmarks, $((1 - \omega) \cdot TIPS + \omega \cdot PSP, \omega \in \{20\%, 30\%, 40\%, 50\%\})$.

Severe shortfall probabilities (see Panel C of Exhibit 10) also decrease substantially when compared to the case in which the LHP was represented by the TIPS portfolio (see Panel B of Exhibit 6).

Exhibit 11 combines the risk and return perspectives by plotting the reduction in the probability-of-a-deficit and probability-of-a-severe-deficit indicators when shifting from the standard LHP (100% TIPS) to the enhanced LHP while maintaining the same level of mean funding ratio. In this analysis, we consider a base case of 40% investment in the PSP and 60% in TIPS, and show the reduction of the required PSP allocation obtained by an investor willing to substitute the enhanced LHP (when either 5% alternative investments or 10% alternative investments are introduced) to the pure 100% TIPS LHP. For instance, we find that when the investment horizon is 20 years, enhancing the LHP by the introduction of 5% (10%) of the real estate and commodities portfolio allows a reduced allocation to the PSP by 14% (31%) while maintaining the mean funding ratio at the same level as with the non-enhanced LHP. The graphs further show the resulting percentage reduction in shortfall probability and expected shortfall. Again for an investment horizon of 20 years, the introduction of 5% (10%) of alternatives (real estate and commodities) within the LHP leads to a 19% (39%) reduction in shortfall probability. The reduction in severe shortfall probability is even greater and reaches a spectacular 42% (78%). Of course, these estimated shortfall probabilities are strongly dependent upon the assumption of normality for the innovations, and they should not be taken at face value. They provide, however, useful indications with respect to how incorporating alternatives in the liability-hedging portfolio could improve ALM risk budgets.

EXHIBIT 9

Return Equivalents with 10% AI: PSP Allocation Reduction



Note: The graph shows the impact of enhancing the LHP beyond TIPS. The enhanced LHP consists of 90% TIPS and 10% alternative investments (see Exhibit 3). The graph yields the required allocations to the PSP to obtain, on average, the same amount of return as in the case of the traditional LHP (TIPS). Various non-enhanced portfolios are taken as benchmarks, $((1 - \omega) \cdot TIPS + \omega \cdot PSP, \omega \in \{20\%, 30\%, 40\%, 50\%\})$.

CONCLUSION

Based on a suitable econometric framework, we have studied the relationship between inflation-driven liabilities and asset returns on bonds, stocks, commodities, and real estate at various horizons. Our empirical analysis suggests that explicitly accounting for long-term cointegration relationships leads to significant differences in forecasted properties of asset returns in terms of their term structure of risk and correlations with the liabilities. In particular, the liability-hedging potential of commodities seems to be understated by the VAR estimation procedure compared to the case when the presence of long-term cointegration relationships is explicitly accounted for. Our results suggest that novel liability-hedging investment solutions, including commodities and real estate in addition to inflation-linked securities, can be designed to decrease the cost of inflation insurance for long-horizon investors. Such solutions are shown to achieve satisfactory levels of inflation hedging over the long term at a lower cost compared to a solution solely based on TIPS or inflation swaps. Intuitively, the increased expected return potential generated through the introduction of commodities and real estate in addition to TIPS in the LHP allows for a reduced global allocation to the PSP while meeting the global performance expectations and, in turn, allows for better risk management properties.

Our analysis can be extended in several directions and, in particular, would ideally encompass other forms of alternative investments. While we have focused on real estate and commodities, institutional investors

Exhibit 10

Global Portfolio (Enhanced LHP + PSP): ALM Indicators

Panel A: Mean Funding Ratio

	Allocation to PSP (in %)										
Horizon	0	5	10	15	20	25	30	35	40	45	50
3	1.01	1.02	1.02	1.02	1.03	1.03	1.04	1.04	1.04	1.05	1.05
7	1.03	1.04	1.05	1.06	1.07	1.09	1.10	1.11	1.12	1.13	1.14
10	1.04	1.06	1.08	1.09	1.11	1.13	1.14	1.16	1.18	1.19	1.21
15	1.07	1.09	1.12	1.14	1.16	1.18	1.21	1.23	1.25	1.27	1.29
20	1.11	1.14	1.16	1.19	1.22	1.24	1.27	1.30	1.32	1.35	1.37
30	1.22	1.25	1.28	1.31	1.34	1.37	1.40	1.43	1.46	1.49	1.52

Panel B: Probability of Shortfall (FR < 1.0)

	Allocation to PSP (in %)										
Horizon	0	5	10	15	20	25	30	35	40	45	50
3	32.78	31.70	32.28	33.70	34.60	35.32	36.08	36.32	36.58	36.88	37.04
7	22.30	21.04	22.84	24.66	26.34	27.36	28.44	29.36	29.90	30.28	30.66
10	17.48	16.18	18.20	20.28	21.98	23.14	24.06	25.12	26.04	26.60	27.04
15	12.98	10.80	12.02	13.50	14.98	16.02	17.12	18.00	18.70	19.20	20.02
20	9.08	7.32	7.70	8.82	9.90	11.22	12.28	13.10	13.84	14.54	15.34
30	5.20	3.44	3.64	3.86	4.60	5.30	5.92	6.72	7.34	8.08	8.88

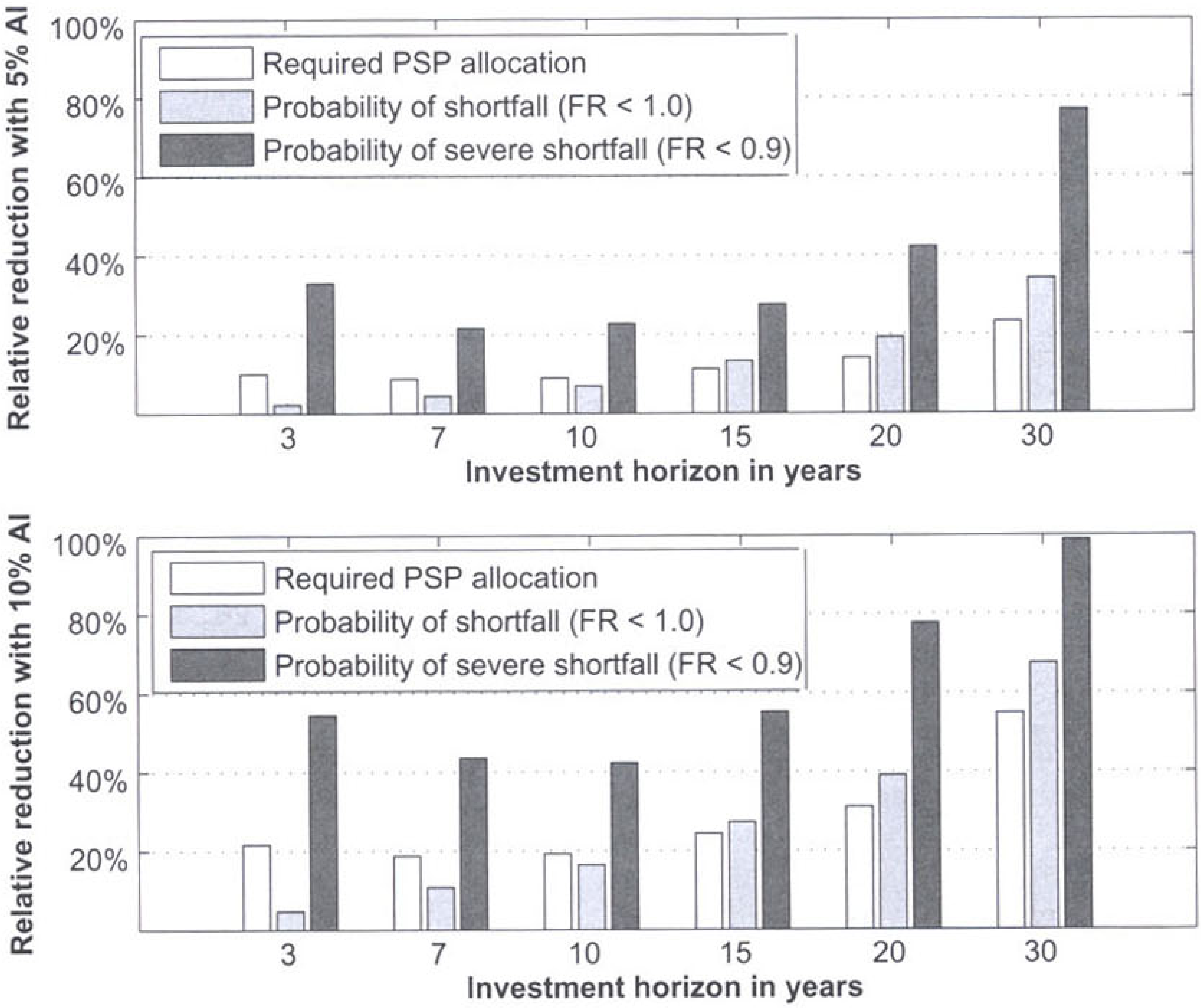
Panel C: Probability of Severe Shortfall (FR < 0.9)

	Allocation to PSP (in %)										
Horizon	0	5	10	15	20	25	30	35	40	45	50
3	0.00	0.00	0.00	0.02	0.16	0.74	1.98	3.60	5.50	7.30	8.82
7	0.00	0.00	0.02	0.16	1.02	2.72	4.80	7.12	9.52	11.32	13.30
10	0.00	0.00	0.02	0.30	1.46	3.26	5.02	7.22	9.38	10.88	12.56
15	0.00	0.00	0.00	0.32	1.14	2.14	3.40	4.84	6.24	7.74	8.94
20	0.00	0.00	0.02	0.16	0.46	1.00	1.76	2.90	4.10	5.20	6.08
30	0.00	0.00	0.00	0.00	0.14	0.20	0.56	1.18	1.72	2.40	2.92

Note: Liability-hedging capacity is examined for portfolios made up of the LHP (TIPS) enhanced by 10% of alternative investments according to Exhibit 3 and the PSP (stocks and bonds) according to the allocation that maximizes Sharpe ratio. Mean funding ratios and shortfall probabilities (in percent) for various investment horizons (in years) and based on 5,000 simulated scenarios are given.

have recently shown an increasing interest in other alternatives, such as private equity and infrastructures, for which intuition suggests that attractive inflation-hedging properties could also be obtained. The unavailability of time series for these asset classes with the sufficient length and frequency is, however, a serious concern from the econometric perspective. One possible solution would involve the construction of liquid proxies for the returns on these assets based on publicly traded instruments with similar characteristics, but the adequacy between the proxy and the actual form of investment under consideration (private equity or infrastructure) would have to be carefully assessed. More generally, it should be emphasized that the search for long-term cointegration relationships should not be handled as a purely statistical exercise that would likely generate spurious results; it should instead be motivated by a careful prior economic analysis of the reasons why inflation and prices for a given asset class might move together in the long run.

EXHIBIT 11
Risk Reduction through LHP Enhancements



Note: The graphs show the impact of enhancing the LHP beyond TIPS on various risk parameters. We considered the base case of an investment of 40% in the PSP and 60% in the traditional LHP and compared this portfolio to the case where the LHP is enhanced by 5% and 10%, respectively, of alternative assets (AI), (i.e., estate and commodities). Enhanced and traditional portfolios are linked such that the portfolios exhibit the same mean funding ratios.

APPENDIX

Model-Implied Returns, Variances, and Correlations

As in continuous-time models, our econometric approach allows us to obtain analytical expressions for time-dependent variances, covariances, and expected returns. To see this, we first write the model-implied forward-looking returns for the first three dates as functions of the interim shocks and current values for y_t and Δy_t,[8]

$$\begin{aligned}
\Delta y_{t+1} &= \Pi y_t + \Gamma \Delta y_t + u_{t+1} \\
\Delta y_{t+2} &= \Pi y_{t+1} + \Gamma \Delta y_{t+1} + u_{t+2} \\
&= \Pi (y_t + \Delta y_{t+1}) + \Gamma \Delta y_{t+1} + u_{t+2} \\
&= \Pi y_t + (\Pi + \Gamma)(\Pi y_t + \Gamma \Delta y_t + u_{t+1}) + u_{t+2} \\
&= \Pi (I + \Pi)\, y_t + (\Pi + \Gamma)\,\Gamma \Delta y_t + (\Pi + \Gamma) u_{t+1} + u_{t+2} \\
\Delta y_{t+3} &= \Pi y_{t+2} + \Gamma \Delta y_{t+2} + u_{t+3} \\
&= \Pi (y_t + \Delta y_{t+1} + \Delta y_{t+2}) + \Gamma \Delta y_{t+2} + u_{t+3} \\
&= [\Pi + \Pi^2 + (\Pi + \Gamma)\, \Pi (I + \Pi + \Gamma)] y_t \\
&\quad + [\Gamma(\Pi + (\Pi + \Gamma)^2)] \Delta y_t + [\Pi + (\Pi + \Gamma)^2)] u_{t+1} \\
&\quad + (\Pi + \Gamma) u_{t+2} + u_{t+3}
\end{aligned} \tag{A-1}$$

We subsequently obtain all finite forward-looking implied returns through iteration and denote

$$\Delta y_{t+k} = \Psi_0^{(k)} y_t + \Psi_\Delta^{(k)} \Delta y_t + \sum_{j=0}^{k=1} \Psi_u^{(j)} u_{t+k-j} \tag{A-2}$$

These expressions may then be used to define model-implied expected returns, variances, and covariances. Using $y_{t+k} = y_0 + \Delta y_{t+1} + \cdots + \Delta y_{t+k}$, it is straightforward to show that we obtain

$$\mathrm{E}_t(y_{t+k}) = \left[\sum_{i=1}^{k} \Psi_0^{(i)}\right] y_t + \left[\sum_{i=1}^{k} \Psi_\Delta^{(i)}\right] \Delta y_t \tag{A-3}$$

$$\mathrm{Var}_t(y_{t+k}) = \sum_{i=1}^{k} \left[\left(\sum_{j=0}^{i-1} \Psi_\varepsilon^{(j)} \right) \Sigma_u \left(\sum_{j=0}^{i-1} \Psi_u^{(j)} \right)' \right] \tag{A-4}$$

where Σ_u denotes the time-invariant covariance matrix of the innovation process u. Correlation coefficients may also directly be derived from Equation (A-4).[9]

ENDNOTES

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[1]The VECM approach has been used in modeling price and return dependencies of financial securities (e.g., Blanco, Brennan, and Marsh [2005] or Durre and Giot [2007]) and, more recently, in inflation-hedging contexts (Westerheide [2006] or Hoesli, Lizieri, and MacGregor [2007]).

[2]More details on distributional assumptions and asymptotic properties of the estimation method can be found in Lütkepohl [2008].

[3]The series is downloadable from Kenneth French's website (borrowed from Ibbotson Associates).

[4]See http://research.stlouisfed.org/fred2.

[5]Incorporating additional features, such as actuarial uncertainty and inflation indexation, would not impact the main message of the article, which focuses on the inflation-hedging properties of real assets.

[6]Note that Kothari and Shanken [2004] also considered the case of a 50-basis points per year inflation premium.

[7]Hoevenaars et al. [2008] used 17 years as the duration of the liability portfolio. To avoid having to rely on interpolation, we used the observable interest rate series from the Fed, which is exclusively available for the maturities of 1, 3, 5, 6, 7, 10, and 20 years (see http://research.stlouisfed.org/fred2).

[8]We use a model with one lag because we will focus on this specification for our later numerical application. Further, as noted in Campbell and Viceira [2004], each VAR(p) model may be transformed into a VAR(1) model by adding a lagged version of the vector of endogenous variables as additional state variables.

[9]Note that similar expressions may be obtained for the VAR model on log returns. In this case, $\Psi_0^{(s)}$ is equal to 0, and $\Psi_\Delta^{(s)}$ and $\Psi_u^{(s)}$ simplify to $\Psi_\Delta^{(s)} = \Psi_u^{(s)} = \Phi_s$. It is obvious that VAR-implied conditional moments are different from VECM-implied moments, except if $\Pi = 0$ and $\Gamma = A$. Note further that the nonstochastic constant term has been introduced as one of the endogenous variables for tractability of the formulas. Accordingly, assuming the first entry is the constant term, the first line and the first column of Σ solely contain zeros.

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